

Warm-up!

x	-4	-2	0	2	4	6	7	10	12
$f(x)$	3	-4	5	8	10	12	13	16	10

Unit 1 Day 2 (1.1+1.4):

- Concavity
- Points of Inflection

HW: Concavity Worksheet

Find the average rate of change of $f(x)$ over the interval $[-2, 2]$

$$\frac{f(2) - f(-2)}{(2) - (-2)} = \frac{8 + 4}{2 + 2} = 3$$

For each interval listed below, decide if it is possible that the rate of change of $f(x)$ is constant over that interval?

~~$-4 \leq x \leq 0$~~

~~$0 \leq x \leq 4$~~

$2 \leq x \leq 6$

$6 \leq x \leq 10$

Give an interval over which the average rate of change of $f(x)$ is zero.

$4 \leq x \leq 12$

$$\frac{f(12) - f(4)}{12 - 4} = 0$$

Same Table, Different Questions

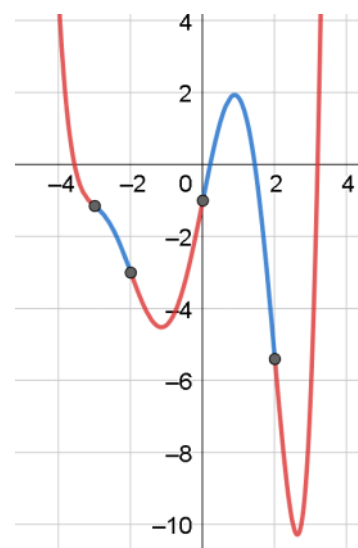
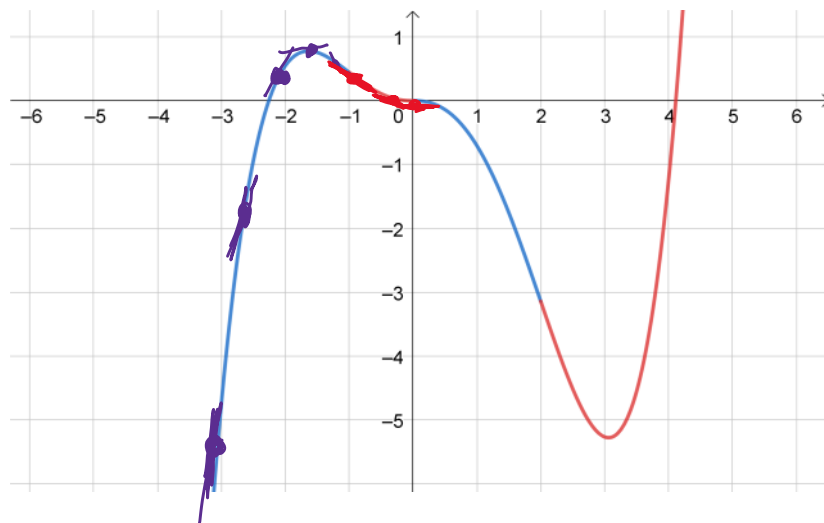
x	-4	-2	0	2	4	6	7	10	12
$f(x)$	3	-4	5	8	10	12	13	16	10

What would be a reasonable guess for the rate of change of $f(x)$ at $x = 1$?

Put the following in order from least to greatest:

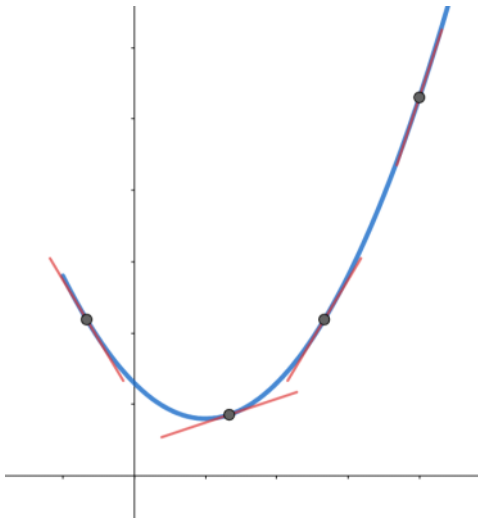
- Average rate of change on the interval $-4 \leq x \leq -2$
- Average rate of change on the interval $-2 \leq x \leq 10$
- Average rate of change on the interval $10 \leq x \leq 12$

We need to further develop our "describing functions" toolset

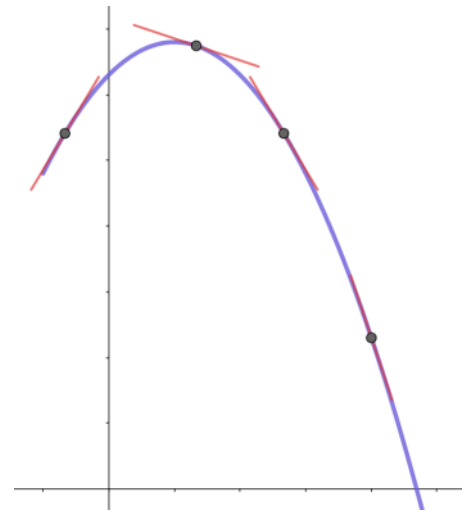


Consider these two graphs.

How would you describe the difference between the red parts and the blue parts?

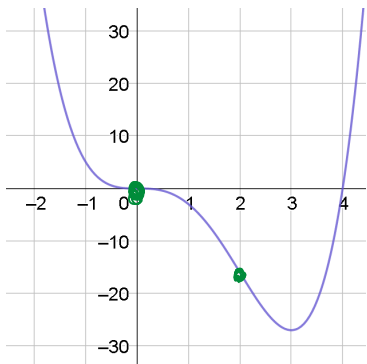


Concave Up
 Looks Like a Cup
 Rate of Change Going Up
 AROC Going Up

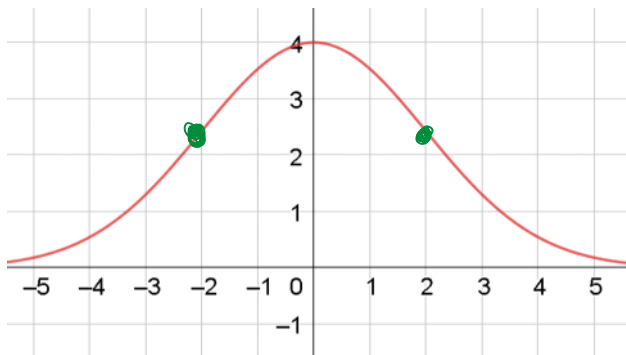


Concave Down
 Looks Like a Frown
 Rate of Change Going Down
 AROC Going Down

Example 1 - Estimate the interval(s) over which each graph is concave up and the interval(s) over which each graph is concave down



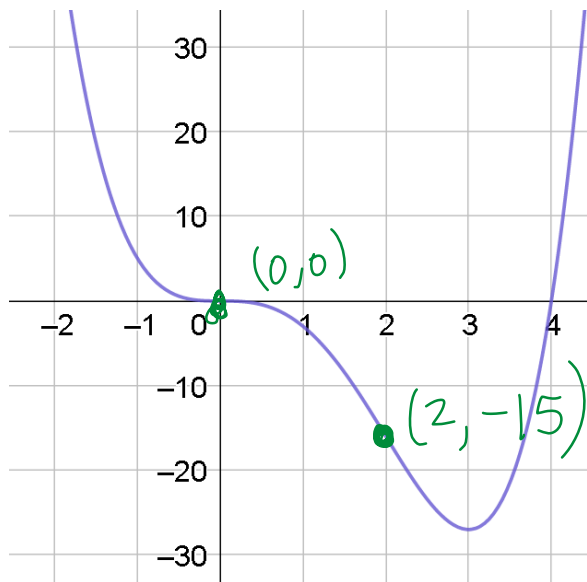
concave up $(-\infty, 0)$
 concave down $(0, 2)$
 concave up $(2, \infty)$



concave up $(-\infty, -2)$ and $(2, \infty)$
 concave down $(-2, 2)$

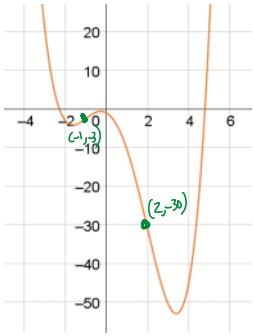
(Note: The other way this question could be asked and mean the exact same thing is "Estimate the interval(s) over which the rate of change is increasing and the interval(s) over which the rate of change is decreasing")

Points of Inflection

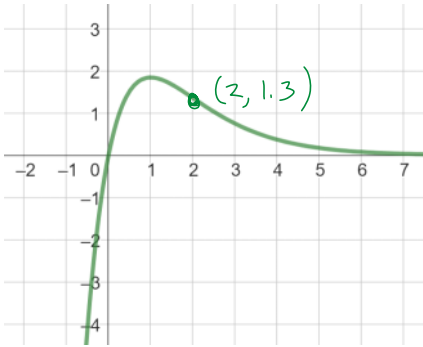


Definition: A point at which the concavity of a graph changes from upward to downward (or downward to upward).

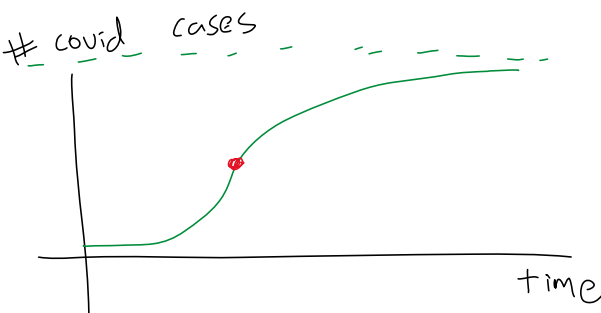
Example 2 - Estimate the intervals over which each graph is concave up/down. Also, estimate the coordinates of any points of inflection



concave up: $(-\infty, -1)$ and $(2, \infty)$
 concave down: $(-1, 2)$



concave up: $(2, \infty)$
 concave down: $(-\infty, 2)$



Example 3: Consider the functions in the following tables.

$$m = 10 \quad m = 6 \quad m = 1 \quad m = 1$$

$$m = 4 \quad m = 6 \quad m = 10 \quad m = 14$$

x	$f(x)$
-5	10
-3	30
0	48
2	50
4	52

Handwritten annotations for $f(x)$:
 - Green arrows: x from -5 to -3 (+2), $f(x)$ from 10 to 30 (+20).
 - Orange arrows: x from -3 to 0 (+3), $f(x)$ from 30 to 48 (+18).
 - Blue arrows: x from 0 to 2 (+2), $f(x)$ from 48 to 50 (+2).
 - Purple arrows: x from 2 to 4 (+2), $f(x)$ from 50 to 52 (+2).

x	$g(x)$
-5	2
-3	12
0	24
2	46
4	70

x	$h(x)$
-5	-4
-3	4
0	22
2	42
4	70

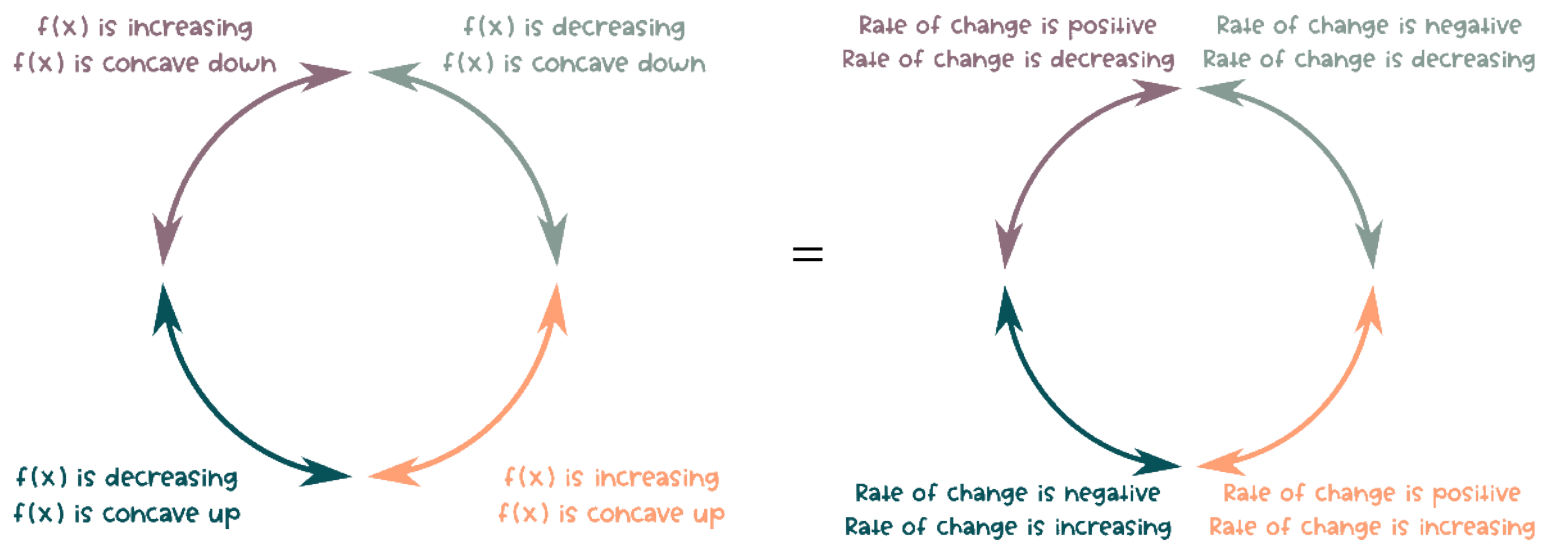
Handwritten annotations for $h(x)$:
 - Green arrows: x from -5 to -3 (+2), $h(x)$ from -4 to 4 (+8).
 - Orange arrows: x from -3 to 0 (+3), $h(x)$ from 4 to 22 (+18).
 - Blue arrows: x from 0 to 2 (+2), $h(x)$ from 22 to 42 (+20).
 - Purple arrows: x from 2 to 4 (+2), $h(x)$ from 42 to 70 (+28).

x	$k(x)$
-5	-15
-3	-5
0	10
2	20
4	30

A

- Which table shows a function that is possibly concave up on the interval $(-\infty, \infty)$?
- Which table shows a function with a rate of change that seems to be positive and decreasing?
- Which table shows a function that seems to be linear?

Pieces of Graph Come in 4 Flavors



Example 4: Over what interval is $f(x)$...

ROC positive and increasing
Increasing and concave up?

$(-1, \infty)$ $f(x)$ increasing
at an increasing rate

Decreasing and concave down?

$(-5, -3)$

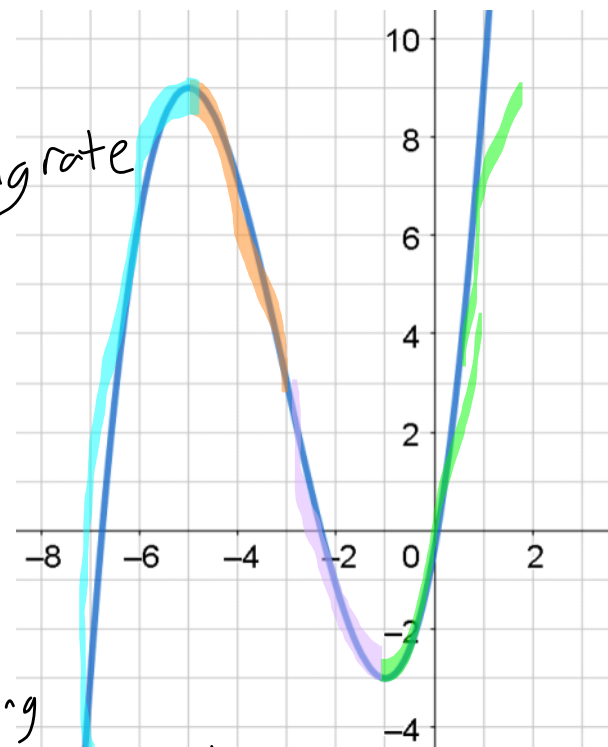
Increasing and concave down?

$(-\infty, -5)$

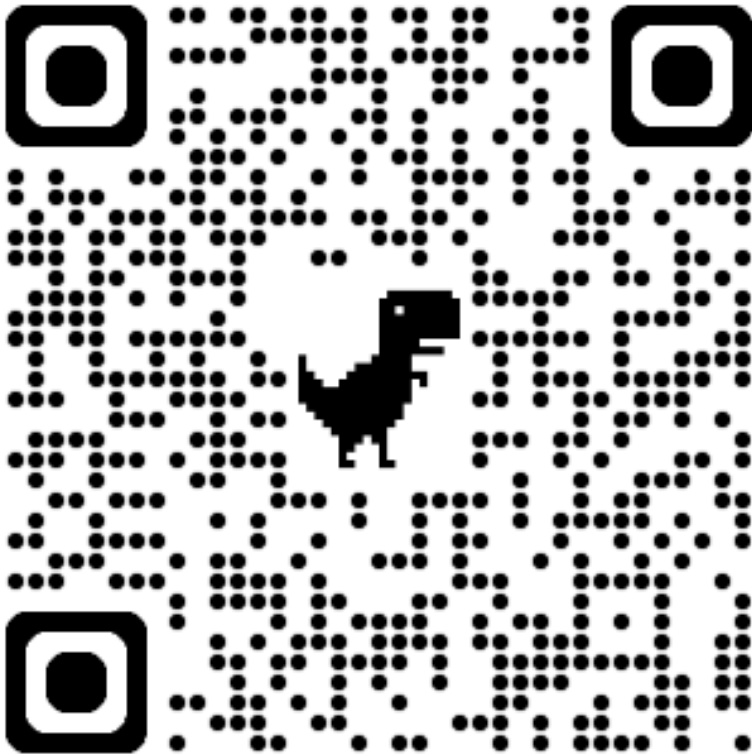
$f(x)$ Decreasing and Concave up?

$(-3, -1)$

$f(x)$ is decreasing
at an increasing rate



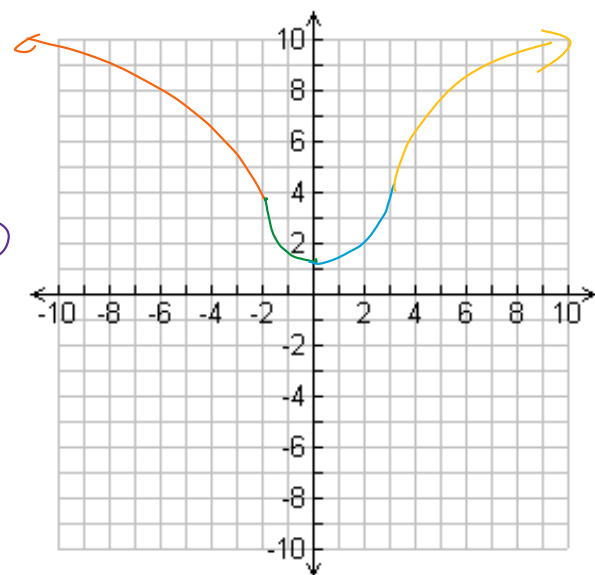
F Vocab Practice: Level 1



Example 5: Sketching

Sketch a continuous function $f(x)$ with the following properties:

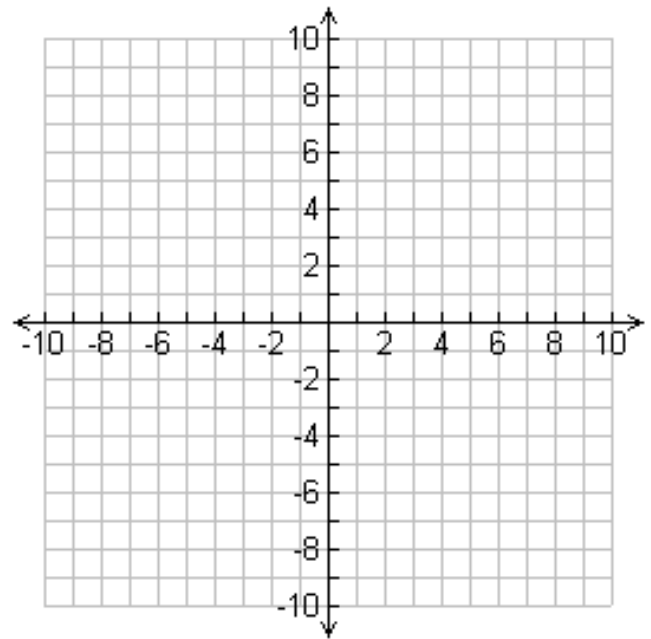
- f is decreasing and concave down on the interval $(-\infty, -2)$
- f is decreasing and concave up on the interval $(-2, 0)$
- f is increasing and concave up on the interval $(0, 3)$
- f is increasing and concave down on the interval $(3, \infty)$



Example 6: Sketching

Sketch a continuous function $f(x)$ with the following properties:

- Increasing at an increasing rate on $(-\infty, 1)$
- Increasing at a decreasing rate on $(1, 3)$
- Decreasing at a decreasing rate on $(3, \infty)$
- The absolute maximum of $f(x)$ is 6



Example 7: Over what interval(s) is $g(x)$...

Decreasing with an increasing rate of change?

$(-\infty, 0)$ and $(2, 3)$

Concave up with a positive rate of change?

$(3, \infty)$

Decreasing with a decreasing rate of change?

$(0, 2)$

Increasing and concave down?

Nowhere

